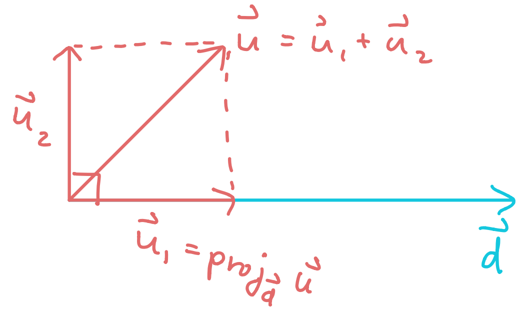
Final Exam study guide

Note: For earlier material, see midterm study guides.

4.2 - Projections + planes

- Dot product: definition and basic properties
- $\vec{v} \cdot \vec{w} = 0 \iff \vec{v}, \vec{w}$ are orthogonal
- Projection - you should know the formula for this



- planes: formula + its meaning
- cross product: know how to calculate this + its geometric significance.

- calculate distances:
  - point to plane
  - plane to plane
  - line to line (parallel vs. not)
  - plane to parallel line

5.1 - Subspaces of $\mathbb{R}^n$

- Definition, how to check if something is a subspace

- nullspace/image space definitions
- eigenspace
- spanning set
- subspaces of $\mathbb{R}^2, \mathbb{R}^3$?

5.2 - Linear independence + dimension

- Definition of linear independence
- $\{\vec{x}_1, \dots, \vec{x}_n\}$ lin. indep $\Rightarrow$ Each vector in span$\{\vec{x}_1, \dots, \vec{x}_n\}$ can be uniquely written as a linear combination of the $\vec{x}_i$.
- How to check for linear independence.
- Def of a basis / dimension
- $n \times n$ matrix invertible $\Leftrightarrow$ cols form a basis for $\mathbb{R}^n$
  $\Leftrightarrow$ rows form basis for $\mathbb{R}^n$
- Any lin. indep. set of vectors can be enlarged to a basis for a subspace U.
- Any spanning set of U can be cut down to a basis.
- If U is contained in V, $\dim U \leq \dim V$ ($=$ if $U=V$).
- How to find dimension/basis for a subspace.

5.3 — Dot product, length, distance, orthogonality

- length, dot product, distance definition + properties.
- Cauchy inequality, triangle inequality
- orthogonal sets
- every orthogonal set is lin. independent
- orthonormal sets

5.4 - Rank of a matrix

- column space / row space definitions
- row operations don't affect row(A)
- how to find basis for row(A) / col(A)
- dim of row/col space = rank (A)
- col (A) = im (A)
- dim (null (A)) = (# of cols) − rank(A)

5.5 - Similarity / diagonalization

- def of $A \sim B$

- similar matrices have same: det, trace, rank, etc.

- Eigenspaces: basis consists of basic eigenvalues.

- A diagonalizable $\iff$ $\mathbb{R}^n$ has a basis consisting of eigenvectors of A
$\iff$ For each λ, $\dim E_\lambda(A) = \text{mult}(\lambda)$.

6.1 - Abstract vector spaces

- definition

- some examples: Polynomials, matrices, real-valued functions